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A Review of Mathematical Models for ESPs and Comparison of Their Successes

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A REVIEW OF MATHEMATICAL MODELS FOR ESPs AND COMPARISON OF THEIR SUCCESSES

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ABSTRACT

Mathematical models of ESP performance exist in many variations, from simple analytical expressions to complex numerical calculations. This paper examines several published ESP models--the Deutsch-Anderson, Matts-Ohnfeldt, Cooperman, Robinson, Southern Research, and RTI sectional models--and compares their predictions of ESP performance against measured data. The analysis shows the common areas of all the models and the specific vagaries of each. In all current models, it is seen that unmodeled effects can dominate the accuracy of the modeling process but that a "calibration" of a given model can make the model useful for those effects it handles well. The values of important parameters for each model are tabulated so that the reader may use them as summary descriptions of the ESP data.

INTRODUCTION

Models for electrostatic precipitator collection performance have been proposed over the years to account for a wide variety of effects observed in ESPs. These models range from the simplest analytical expressions to complex numerical simulations, yet all have implicit or explicit assumptions glossing over unmodeled effects. The unmodeled effects often appear as adjustable parameters whose values are determined by experience with a variety of operating ESPs, but this approach leaves the model user open to criticism when extrapolating beyond the realm of experience.

An important aspect of modeling is to replace complex phenomena by average values when that replacement will have little effect on the results of the model. An example of this is the use of the SCA (specific collector area) concept in ESP evaluations. This parameter neglects such factors as height or width of lanes, baffles on collector plates, turbulence, velocity distributions, and many others. Yet, within the framework in which it is used, it represents a controlling parameter on ESP performance.

The models considered in this paper will be termed "complete" models because they all explicitly allow the precipitator performance to be evaluated. There are many ESP-related models, such as turbulent diffusion models, which address part of the precipitation process (collection, for instance) but not the whole process from inlet to outlet. The complete models may require auxiliary information in calculating factors, but all necessary factors are included.

The models considered here all evaluate the mass collection performance of ESPs; one of the models provides enough additional information to evaluate the collection performance for fine particles and, thus, predict the stack opacity. It should be possible to modify all the models to the same end but only at the expense of greatly increased complexity. Therefore, all comparisons of the models will be done on a mass collection basis.

All the models considered in this work have adjustable parameters whose values cannot be determined from theory. Although some of these parameters have commonly accepted values which prove useful in evaluating ESPs, they will be treated as fitting parameters in testing the models against measured performances. The functional form of the model determines the sensitivity of the model to the values of the fitting parameter(s); the parameter is required to absorb all the deviations due to unmodeled effects; or, if a single value is used, the fit to the data will deteriorate.

A figure-of-merit parameter will be used to compare the various models. It will measure

the deviations of the models' predictions from a standard experimental data set and will be described in more detail later. Although it is not intended to be a sensitive indicator of model performance, the figure of merit does show some strong differences between the models.

DESCRIPTION OF THE MODELS

The six models can be described mathematically in terms of the predicted mass penetration through the ESP. The more complex models often require auxiliary calculations to arrive at appropriate values for some of the quantities that appear in the penetration expression, but these calculations are implicit in the model and could be replaced by experimental data or other theoretical expressions.

Deutsch-Anderson Model

The Deutsch-Anderson (DA) model [1] describes the penetration of the ESP in terms of a relative plate area (the SCA) and a single parameter termed the migration velocity. Because it was the first complete ESP model, most discussions of ESP performance refer to it.

$$p = \exp(- w_e \, SCA). \tag{1}$$

Here, p is the mass penetration, w_e denotes the effective migration velocity, and SCA is the plate area per unit volume of gas treated. The w_e is usually evaluated empirically, but it can be expressed in terms of a theoretical particle velocity.

Two approaches will be taken to evaluate w_e: the first is a power law fit to the existing data set, and the second is a fractional multiplier of the theoretical drift velocity. These are written explicitly as:

$$w_e = w_{avg} \, (SCA_{avg}/SCA)^u \tag{1a}$$

and

$$w_e = a_e \, w, \tag{1b}$$

where w_{avg} is a fitted mean value of migration velocity, SCA_{avg} is a fitted average SCA, u is the power law exponent, a_e is a fitting constant, and w is the theoretical drift velocity. The theoretical drift velocity is considered in all the models and appears explicitly in several. A common method for evaluating w from basic EPA information will be described.

Matts-Ohnfeldt Model

The Matts-Ohnfeldt (MO) model [2] is a modification of the DA model, intended to account for variations in particle distributions for ESPs of nearly equivalent size:

$$p = \exp[- (w_k \, SCA)^k]. \tag{2}$$

Here, w_k is the effective MO migration velocity. The exponential factor, k, is often allowed to vary, but common practice assigns it the fixed value 0.5. Both cases will be considered in this comparison; if the exponent is equal to 1.0, the MO model is identical to the DA model. Like the DA model, the migration velocity will be calculated in two ways:

$$w_k = w_{avg} \, (SCA_{avg}/SCA)^z \tag{2a}$$

and

$$w_k = a_k \, w, \tag{2b}$$

where w_{avg} is a fitted mean value of migration velocity, SCA_{avg} is a fitted average SCA, z is the power law exponent, a_k is a fitting constant, and w is the theoretical drift velocity.

Cooperman Model

The Cooperman (CC) model [3] uses two factors to modify the theoretical drift velocity, as applied in the DA relation:

$$p = \exp(- w_c \, SCA) \tag{3}$$

with

$$w_c = v[(a^2 + 2b)^{0.5} - a], \tag{4}$$

$$a = vd/2D, \tag{5}$$

and

$$b = (1-f)dw/2D. \tag{6}$$

Here, w_c is the Cooperman migration velocity, v is the gas flow velocity through the ESP, d is the half width of the duct, w is the theoretical drift velocity, and f and D are parameters for the model. Originally, f was assumed to represent the ratio of particles moving away from the walls to those moving toward the walls, and D represented a diffusivity coefficient, but these interpretations have been disputed.

Robinson Model

The Robinson (MR) model [4] was formulated as a three-parameter model to include effects of nonuniform particle concentration and two types of erosion of collected particles. The resulting collection equation reduces to a two-parameter form:

$$p = (1 - B) \exp(- A \, w \, SCA) + B \tag{7}$$

with

$$A = X(1 - a) \tag{8}$$

and

$$B = b(1 - a). \tag{9}$$

Here, w is again the theoretical drift velocity. The parameter X is the ratio of the particle concentration at the wall of the ESP to its mean value, a is the mass of dust eroded per mass of dust collected, and b is the mass of dust eroded per mass of dust at the inlet.

Although a, b, and X have theoretical interpretations, there is no way to evaluate them directly, and the parameters A and B will be used instead.

RTI Sectional Model

The Research Triangle Institute (RTI) model [5] was formulated as an analytic model to deal explicitly with sneakage and rapping reentrainment losses. It is the first model in this comparison to require inputs and calculations for each electrical section of the ESP. The models described before may be used in such fashion but lose their simplicity in such use. Moreover, there is no prescribed method for "sectionalizing" those models. The RTI model makes such sectionalizing explicit.

The RTI model is a two-parameter model, with explicit assumptions about the nature of rapping emissions, particle collection, and back corona. Such assumptions may limit its applicability but no more so than any of the other models.

The total penetration is the product of the individual penetrations of the sections.

$$P_t = \prod_{i=1}^{n} P_i, \tag{10}$$

where n is the number of sections, and P_i is the penetration through the ith section. The section penetration is similar to Robinson's model:

$$P_i = (1 - L) \exp(- w_i \, SCA_i) + L, \tag{11}$$

where w is the theoretical drift velocity, and L is the loss factor for the section, given by:

$$L = SN + RR - SN \cdot RR. \tag{12}$$

The parameters SN and RR stand for sneakage and rapping reentrainment fractions, respectively, and are assumed to be constant through the ESP. Although at this stage the RTI model appears to use only one parameter, L, the treatment of the theoretical drift velocity requires separate sneakage and rapping parameters.

The theoretical drift velocity is calculated from the mass median diameter (MMD) of the particle distribution entering a given section. The MMD is given as a mass weighted average of three terms: the uncollected fraction emerging from the prior section, the fraction sneaking by the prior section, and the fraction reentrained by rapping from the prior section.

$$MMD_i = \frac{m_o MMD_o + m_s MMD_s + m_r MMD_r}{m_o + m_s + m_r}, \tag{13}$$

where the i subscript stands for the inlet to a section, o stands for the outlet of the prior section, s stands for sneakage, r stands for rapping, MMD for the characteristic diameter of each type of emission, and m for mass concentration.

The characteristic particle sizes are as follow. The rapping MMD is taken to be 6 μm [6], the sneakage MMD is the same as the MMD at the inlet of the prior stage (since it represents material which bypasses that stage), and the uncollected fraction MMD is taken as a penetration weighted interpolation between the inlet MMD of the prior stage and 3 μm. This last is expressed as:

$$MMD_o = p_c \, MMD_i + (1 - p_c)3 \tag{14}$$

where p_c is the Deutsch-Anderson penetration of the section, with an inlet MMD_i.

SRI Model

The Southern Research Institute (SRI) model [7] is a full computer model which calculates particle charging and electrical conditions from theoretical expressions and applies them to the collection of particles in discrete size bands. In spite of the complexity of the model, the collection process can be described in fairly simple equations, assuming all the numerical processes have converged.

First, the Deutsch-Anderson collection process is applied over a series of small increments for each particle size band, using the local electrical conditions, to obtain an ideal penetration for the whole ESP:

$$P_I = \sum_{j=1}^{l} \prod_{i=1}^{m} \exp(- w_{ji} \, SCA_i), \tag{15}$$

where P_I is the ideal penetration, j is an index for particle sizes, and i is an index for incremental lengths. The theoretical drift velocities are used for w_{ji}, with no allowance for back corona.

A series of corrections are applied as multipliers of the theoretical drift

velocities, dependent on the ideal penetration, and produce a corrected penetration:

$$P_C = \sum_{j=1}^{l} \sum_{i=1}^{m} \exp(- F_j w_j SCA_i), \qquad (16)$$

where P_C is the corrected penetration, and F_j is a size dependent correction for drift velocity, sneakage, and velocity maldistribution effects.

The rapping reentrainment correction is based on a correlation of field measurements [6] and adds a predetermined particle distribution to the penetration, with a mass fraction dependent on the corrected penetration:

$$P_R = P_C + G(P_C) \sum_{j=1}^{l} (m_r/m_I)_j, \qquad (17)$$

where P_R is the rap-corrected penetration, G is the fraction of emissions due to rapping, and m_r/m_I is the ratio of rapping mass concentration to inlet mass concentration.

Since the factor G is determined in the model, the only adjustable parameters are for sneakage, SN, and velocity maldistribution, s_g, both of which appear compositely in the factor F but with different effects.

Theoretical Drift Velocity

The theoretical drift velocity is not one of the migration velocities, which are parameters for performance, but the actual electrically driven motion of the particle relative to the gas stream. Its value is commonly given by Stokes' law:

$$w = q\ C\ E/6\pi\ \eta\ r, \qquad (18)$$

where q is the charge on the particle, C is the Cunningham correction factor, E is the electric field, η is the gas viscosity, and r is the particle radius. For large particles and high velocities, Stokes' law itself requires correction terms which will be neglected here.

For these models, the theoretical drift velocity must be fixed according to some rational method. Accordingly, the appropriate particle diameter was chosen to be the MMD of the particle distribution, since that information is usually available. Since the MMD is greater than 5 to 10 μm for the ESPs considered, it is justifiable to use the field charging mechanism to describe the charge on the particle, with the particles assumed to be conductive. The electric field to be used in the calculations is that in the first electrical section for whole ESP models, or in each electrical section for the sectional models. The gas viscosity is that of air, corrected for temperature, and the Cunningham factor is usually neglected.

The resulting relation for the particle drift velocity is:

$$w = 2M\ \varepsilon_o\ E^2\ r/\eta, \qquad (19)$$

where M is the fraction of charge attained due to back corona, ε_o is the free-space permittivity, and all the other terms are as defined before.

The factor M is a major addition to the drift velocity relation and is based on experimental and theoretical grounds [8]. The theory predicts a reduction in the net charge on the particle in the presence of both positive and negative ions, and many experimental measurements on the ratio of positive to negative current densities have been made. The resulting factor M should be given the value 1.0 when there are no signs of back corona and the value 0.5 when there are obvious signs of back corona (V-I curves, high resistivity dust, sparking at low currents). It is feasible to use intermediate reductions in charge for less obvious cases of back corona, but it is difficult to quantify the value.

EQUIVALENCES BETWEEN MODELS

It is convenient to group the models into two sets of three for comparison. The DA, MO, and CC models fall into a group in which all collection related effects are expressed in a migration velocity. The MR, RTI, and SRI models as a group attempt to account for nonideal effects in explicit ways. If rapping losses were not included, the SRI model would be placed in the first group.

In the first group, the prediction of an ESP's performance amounts to evaluating the migration velocity, assuming the SCA is known. The DA and MO models offer no direct help in this matter, although two approaches are suggested in this paper, while the CC model bases its evaluation on the theoretical drift velocity, the gas velocity, and the plate spacing. As will be seen, the best parameters for the CC model remove the dependence on gas velocity for large scale ESPs.

None of the nonideal effects considered in the second group--sneakage, rapping and nonrapping losses, or velocity maldistribution--appear as factors in the first group. Cooperman's factor f (Equation 6) can be interpreted in a fashion similar to Robinson's factor a, as a reentrainment fraction, but Cooperman repudiated that interpretation. Similarly, the mixing factor D in the CC model has a counterpart in the SRI model factor F_1 (Equation 16) but is not interpreted as a sneakage or maldistribution factor by the authors.

If SCA is considered a constant, then the DA and MO models are completely equivalent because corresponding to a given penetration are unique values for w_a and w_k. The CC model is claimed to be equivalent to the MO model [3] but only for fortuitous values of the parameters. Nonetheless, all three models are very similar in their treatment of ESP collection.

The differences within the first group occur when attempting to adjust an SCA to meet a target penetration, with inherent uncertainties in the migration velocity. Then, assuming a migration velocity is known through some other method, the MO model exhibits much less sensitivity to the parameter values because of the fractional exponent; the CC model can also exhibit less sensitivity, depending on the values of the parameters.

In the second group of models, there is an immediate similarity between the MR model and the RTI model. Equations 7 and 11 have the same form, except for a multiplier for the theoretical drift velocity. The RTI model is a direct descendant of the MR model which explicitly breaks the ESP into sections.

It would be convenient to identify the MR factor B with the RTI factor L raised to the n^{th} power; this is possible only if the theoretical drift velocity is large enough that the exponential terms are nearly zero. Otherwise, the size dependent effects in the RTI model make the comparison invalid. The SRI model, seen in Equations 16 and 17, exhibits a similar functional form to the MR model, but the correspondence in the loss term (1-B) is missing in the SRI model.

Thus, the second group explicitly adds a penetration term for losses to the main term for penetration due to collection. This is the principal feature distinguishing the second group from the first group. The additional penetration term sets a lower limit on the penetration achievable by the ESP. In the first group, any level of penetration is theoretically attainable with enough SCA.

Other parts of the treatment reveal similarities to the first group. The treatment of the theoretical drift velocity in the MR model is most similar to Cooperman's treatment, while the RTI model uses the theoretical velocity alone. The RTI model adjusts the particle size through the sections of the ESP to smaller sizes, meaning that the inlet MMD is effectively multiplied by a reducing factor, if all the sections were referenced to the inlet MMD. In the SRI model, the particle size distribution changes as the collection proceeds down the length of the ESP; this automatically reduces the MMD. Thus, in effect, all four models--CC, MR, RTI, and SRI--predict a reduction in collection from that attained with the pure theoretical drift velocity at the inlet. For the DA and MO models, certain values of the constants in Equations 1b and 2b would also effect a reduction from the theoretical drift velocity.

It is difficult to assess the sensitivity of the second group to changes in their parameters or to variations in SCA. The losses are treated differently in each and represent unmeasured quantities in most cases. In addition, the equations themselves cannot be inverted easily to estimate scatter in parameters.

DATA FOR TESTING MODELS

Data for testing of the models were obtained from several reports of field tests of full-scale ESPs [7,9,10,11], in which sufficient information was presented to run most of the models. In two of the reports [7,10], results of running the SRI model were presented (results slightly different from those in Reference 7 were reported in Reference 12). These were the only evaluations of the SRI model used because of time limitations.

Many of the models require little more than SCA, inlet MMD, and electrical conditions in the first section to allow a prediction of the penetration. The RTI model requires, in addition, section SCAs and electrical conditions. The SRI model requires size distribution data and many details of the mechanical and electrical operation; not all of the details are available in the reports cited, but reasonable estimates may be made with little effect on the results. The full Cooperman model requires the gas velocity, a quantity usually inferred from the volume flow and cross-sectional area but only rarely reported as a measured value; fortunately for this comparison, the parameters made the penetration equation insensitive to gas velocity.

Table 1 contains a partial summary of the data taken from the reports. The ESPs are ranked by SCA; the measured penetrations are tabulated; the numbers of active sections are shown; and the theoretical migration velocities, calculated according to Equation 19, are presented. Several ESPs were measured under conditions of reduced load or reduced number of sections, accounting for the appearance of several designators in more than one place.

The performance of these ESPs is shown in Figure 1, as the logarithm of penetration plotted against the logarithm of SCA. These axes are chosen because of the wide range of values in both parameters. There is only a slight trend in evidence for penetration as a function of SCA, with much scatter of the points around any trend line. This lack of trend immediately suggests that all models in

TABLE 1. ESP DATA FOR MODEL TESTING

ESP [Reference]	SCA (s/m)	Penetration	Number of Sections	w (Theoretical) (cm/s)
George Neal 2 [11]	36	8.90E-03	3	71
EPRI 2 [7]	49	4.50E-03	3	146
EPRI 2 [7]	49	9.70E-03	3	87
EPRI 3 [7]	50	1.05E-03	4	203
EPRI 6 [7]	55	9.80E-03	6	35
EPRI 4 [7]	68	6.60E-03	4	62
Allen [9]	78	7.80E-03	4	26
George Neal 3 [10]	110	1.03E-02	3	31
EPRI 6 [7]	116	7.83E-02	6	12
EPRI 5 [7]	120	1.50E-03	5	40
EPRI 1 [7]	147	7.80E-04	6	13
George Neal 3 [10]	147	3.00E-03	4	31
George Neal 3 [10]	250	1.10E-03	4	31

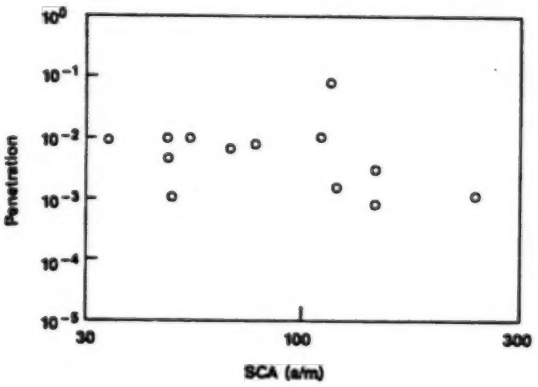

Figure 1. Penetrations measured on full-scale ESPs used for model testing.

which SCA plays a major role will have difficulty in matching the measured performances.

The parameter that was chosen to evaluate the models was the standard deviation of the differences between the logarithms (base 10) of the measured penetrations and the predicted penetrations. The standard deviation was calculated by summing the squares of the differences of the logarithms, dividing by the number of independent data points, and extracting the square root. The number of independent data points was the number of test cases minus the number of fitting parameters. This figure-of-merit parameter has the property of indicating the relative goodness-of-fit of each model over the full SCA range and of compensating for the differences in number of fitting parameters and data between the various models.

The figure of merit would be zero for a model that fits the data perfectly; it would be 0.53 if the experimental average value of penetration were used to predict performance. For convenience, this figure of merit will be assigned to the "Null model," so named because it has no information content.

EVALUATION OF ADJUSTABLE PARAMETERS AND FIGURES OF MERIT

In the models, Deutsch-Anderson and Matts-Ohnfeldt for the migration velocities, the first approach used, the power law fits to SCA, actually transforms the DA and MO models into two-parameter models, with the average migration velocity and the exponents, u or z, as parameters. The constant multiplier representations remain one-parameter models, except in the MO case where the exponent, k, is also allowed to vary for best fit. Thus, the DA and MO model performances have different figures of merit, according to which method is used for the effective migration velocity. The other models have no such ambiguity. There are two sets of results published for the SRI model, but the differences between them are probably not significant.

Table 2 presents the figures of merit for all the models, ranked according to performance, and Table 3 contains the values of the parameters used to achieve those figures. It is immediately apparent that some of the models

have a worse figure of merit than even the Null model.

TABLE 2. FIGURES OF MERIT

Model	Figure of Merit
RTI Sectional	0.14
SRI Numerical [7]	0.33
MO (Variable k, Equation 2b)	0.37
SRI Numerical [12]	0.38
MO (k = 0.5, Equation 2b)	0.39
MR	0.48
NULL	0.53
DA (Equation 1a)	0.56
MO (k = 0.5, Equation 2a)	0.56
CC	0.62
DA (Equation 1b)	0.82

but because back corona was evident, the model could only achieve reasonable results by artificially raising the value of the velocity maldistribution factor, a_g. Equation 16 shows that this is a justifiable way to modify the theoretical drift velocity if no other avenue exists. The error bounds in Table 3 represent one standard deviation in the values for SN and a_g. For a_g, the variation includes the correction for back corona already mentioned.

Since fits for the SRI model covered only a portion of the data and since back corona is not specifically included in the model, the figure of merit and the variability of the parameters might well decrease if these considerations were included. On the other hand, the MO model (variable k, Equation 2b) was tested on the same data set as the SRI model and achieved a figure of merit of 0.28, indicating that the ranking might change if the SRI model had access to all the data.

The fitted DA and MO models show essentially the same figure of merit as the Null model, a situation which indicates that the

TABLE 3. MODEL PARAMETERS

Model	Relevant Equations	Parameters
RTI Sectional	12	SN = 0.07, RR = 0.12
SRI Numerical [7]	16	SN = 0.075 ± 0.034, a_g = 0.36 ± 0.16
MO (Variable k, multiplier)	2, 2b	k = 0.4, a_k = 9.50
SRI Numerical [12]	16	SN = 0.075 ± 0.034, a_g = 0.36 ± 0.16
MO (k = 0.5, multiplier)	2, 2b	k = 0.5, a_k = 4.10
MR	8, 9	A = 0.286, B = 0.0032
NULL	- -	Average penetration = 4.76E-03
DA (power law)	1, 1a	w_{avg} = 0.067 m/s, SCA_{avg} = 81.45 s/m, u = 0.87
MO (k = 0.5, power law)	2, 2a	w_{avg} = 0.37 m/s, SCA_{avg} = 81.45 s/m, z = 0.73
CC	5, 6	D = 20.0, f = 0.0
DA (Equation 1b)	1, 1b	a_e = 0.60

The figure of merit for the SRI model deserves some cautions. The published results [7,12] were not an attempt to establish the parameters as fitting parameters but to observe the accuracy of prediction with "reasonable" values. Therefore, the figure of merit reflects at least some of the error in the prediction ability of the model. In Reference 10, a close fit to the data was obtained; content of the models is low. When the DA and MO models use the theoretical drift velocity with a constant multiplier, the DA model has a worse figure of merit, but it follows the trends in penetration fairly well, as does the CC model. Both the DA and CC models overcompensate the theoretical drift velocity. The MO model has a much better (lower) figure of merit, for the standard value of k, 0.5, and

for k = 0.3 - 0.4. These values of the exponent represent a reduction in the effect of the theoretical drift velocity on the overall penetration.

The MR, SRI, and RTI models achieve the same type of reduction in the effect of the theoretical drift velocity by the inclusion of independent loss terms and, in the latter two models, by the decrease of average particle size through the ESP. As a result, the MR model performs better than the models that do not reduce the effect of drift velocity but not as well as those models that include more of the effect.

It was interesting to note that the first four models considered--DA, MO, CC, and MR--had better figures of merit universally if the inlet MMD was replaced by a constant value of 6 cm; that substitution had the direct effect of reducing the predictions' dependence on the theoretical velocity since all MMDs were larger. The 6-μm value corresponds to the measured MMD of the rapping puff.

The RTI and SRI models achieved agreement on the value of the sneakage parameter, and examination of the rapping reentrainment factor in the SRI model shows that it is close to the fitted value in the RTI model, at least for cold-side ESPs. The loss factor B in the MR model, which represents a penetration lower limit, is consistent with the RTI values for RR and SN, when the number of sections over which the losses occur is included: three or four sections bracket the MR loss factor.

The figure of merit has a statistical interpretation: it is one standard deviation in the prediction error of the base 10 logarithm of the penetration. Therefore, 68 percent of the predictions would lie within 10^{x} of the true penetration, where x is the figure of merit. Thus, for the RTI model, the predictions would be within a factor of 1.4 ($10^{0.14}$) of the true value with 68 percent confidence limits. The performance of each model is shown graphically in Figures 2 through 9, and the figure of merit is represented as a length on each graph.

CONCLUSIONS ABOUT MODELS

Aside from the ranking of the models, the important conclusions that can be drawn from this comparison are the following. First, the more information that is built into the model, the more able it is to make reliable predictions. This is evident in the rankings because the best performing models are those which contain explicit loss terms and allow for the variation of particle size through the ESP. The RTI and SRI models, which include sectional electrical conditions, show improvements over the other models, particularly the MR model which has a similar structure but does not include sectionalization.

Second, particle size effects at the inlet are extremely important for the overall operation of the ESP. All of the models show that including the particle size through the theoretical drift velocity produces penetrations which follow the trends from case to case very well, even for the lowest ranked models. Moreover, the particle size distribution is represented well by the MMD, according to fit to the data. This may be a consequence of the similar distributions observed for the test cases.

Third, the electrical effects, reflected in the theoretical velocity, are also very important but not so obvious. The reason is that most ESP inlet sections run at average electric fields of about 2.5 to 3 kV/cm, not a large range of variation.

Fourth, the inclusion of back corona, by reducing the particle charge, allows the models to predict reduced ESP performance in a way that would be artificially obscured otherwise. The major limitation of the SRI model in this comparison is that there is no provision for reducing particle charge. When that effect is not modeled explicitly, some other parameter must make up the difference in penetration, resulting in more variability and less confidence in the parameter.

Fifth, all the high ranked models show that the theoretical drift velocity has a major, but not total, role in the ESP penetration. Also important are the loss mechanisms however they may be included. The specific loss factors in the RTI, SRI, and MR models add a penetration due to losses; the MO model reduces the effect of the theoretical drift velocity by means of the fractional exponent.

It is apparent from this comparison that the RTI, SRI, and MO models are the most accurate in their predictive capabilities. The Matts-Ohnfeldt model is the most convenient to use but only slightly more so than the RTI model. Both the RTI and SRI models require more information about the sectional aspects of the ESP, which may be difficult to obtain. Given the relatively uniform electrical and loss conditions for ESPs, the most important unknowns for a new design appear to be the particle size information and the presence or absence of back corona. It is also apparent that a relatively simple model, MO, can give results as good as the more complex SRI model, illustrating the point that a calibration of the MO model to a specific type of data (coal fly ash) may allow it to be very useful, even though the model parameters represent averages over many effects.

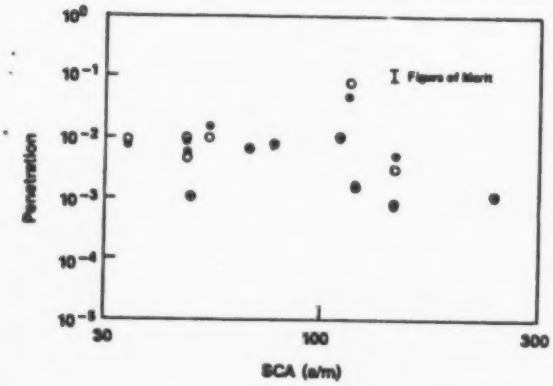

Figure 2. RTI sectional model results (•) compared with measured penetrations.

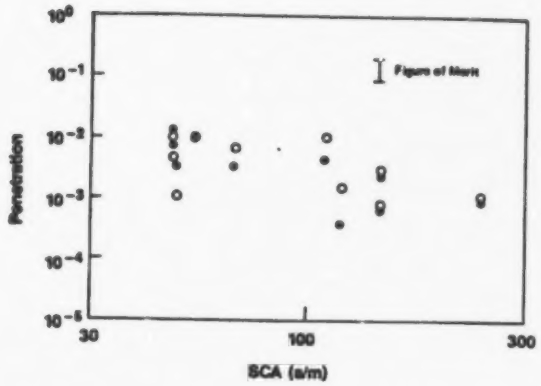

Figure 3. SRI model results (•) compared with measured penetrations (reduced data set).

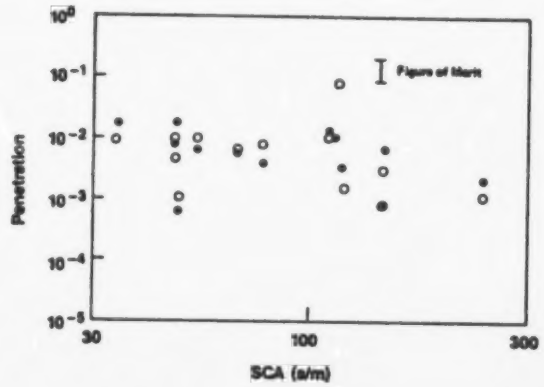

Figure 4. Matts-Ohnfeldt model results (•) compared with measured penetrations (k = 0.4).

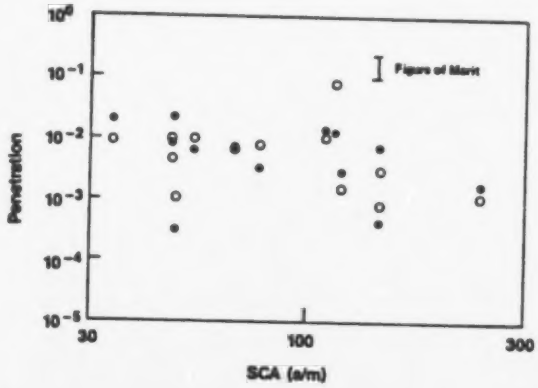

Figure 5. Matts-Ohnfeldt model results (•) compared with measured penetrations (k = 0.5).

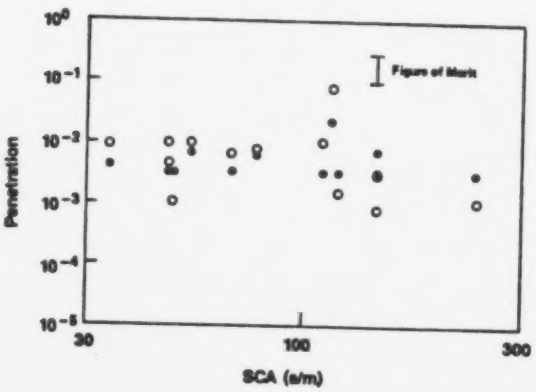

Figure 6. Robinson model (•) compared with measured penetrations.

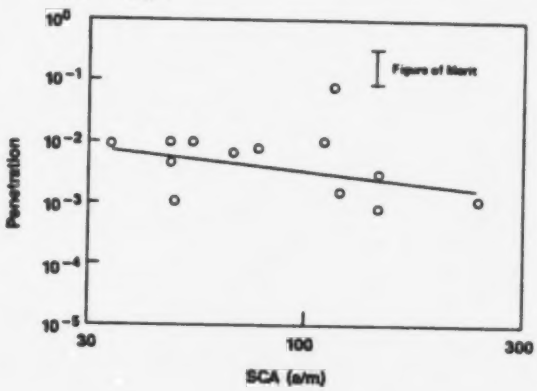

Figure 7. Power law fits of Matts-Ohnfeldt and Deutsch-Anderson models (-) to measured penetrations. (Identical fitting lines.)

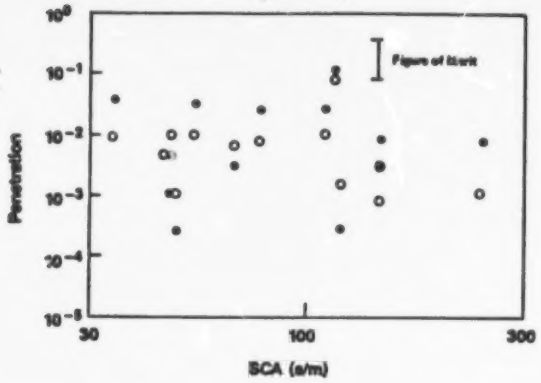

Figure 8. Cooperman model (•) compared with measured penetrations.

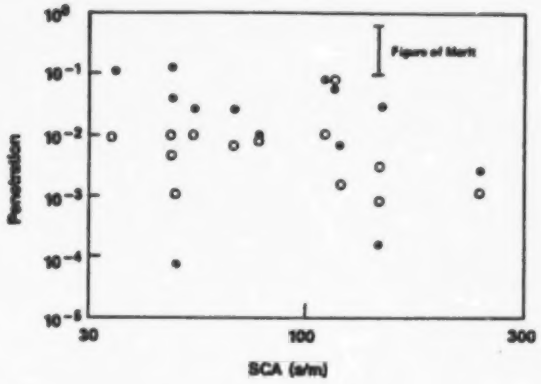

Figure 9. Deutsch-Anderson model (•) compared with measured penetrations.

The RTI model performance shows what can be achieved when the behavior of particle distributions and electrical charging conditions are expressed as appropriate averages. Whether or not the constants in the model allow its application to other dusts, the structure of the model makes the recalibration process much simpler.

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